Dynamic Fine-Grained Localization in Ad-Hoc Networks of Sensors

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Abstract— **Wireless communication systems have become increasingly common because of advances in radio and embedded system technologies. In recent years, a new class of applications that networks these wireless devices together is evolving. A representative of this class that has received considerable attention from the research community is the wireless sensor network. Such a sensor networks consist of numerous tetherless devices that are released into the environment and organize themselves in an ad-hoc fashion. The goal of the network is to perform a monitoring task, and knowledge the physical location of the individual nodes is therefore essential. Not only is this information needed for the sensor network to report the location where events take place, it also assists in group-querying or routing traffic to a designated geographic destination and provides information on physical network coverage. However, equipping every node with a GPS receiver is not always feasible due to possible obstructions in the path of the satellite signals or energy limitations in the nodes. In this paper, we present a novel location discovery approach, which we call AHLoS (Ad-Hoc Localization System), for wireless sensor networks. Only a small fraction of the nodes start with knowledge of their location and the others dynamically estimate their positions via a distributed algorithm. Furthermore, our algorithm utilizes a new iterative multi-lateration technique, such that all nodes that meet some simple connectivity requirements are eventually able to determine their position. We have analyzed the operation of AHLoS, designed a new testbed of wireless sensor nodes and verified the behavior of our distributed localization technique. The results obtained from the testbed are then incorporated in a simulation platform to perform scalability tests and evaluate the effects of error propagation.**

Keywords—**location discovery, localization, wireless sensor networks**

I. INTRODUCTION

A. Sensor Networks and Location Discovery

Nowadays, wireless devices enjoy widespread use in many different settings and the spectrum of applications is rapidly expanding. Ubiquitous connectivity becomes possible by networking these devices together. The exciting new field of *wireless sensor networks* breaks away from the traditional end-to-end communication of voice and data systems, and relies on distributed collaborate information exchange. Myriads of tiny embedded devices, equipped with sensing capabilities, together form an ad-hoc network. Information exchange among collaborating sensors becomes the dominant form of communication, and the network essentially behaves as a large, distributed computation machine. Applications featuring such networked devices are getting increasingly prevalent, ranging from environmental and natural habitat monitoring, to home networking, medical applications and smart battlefields. For example, a sensor node can signal a machine malfunction to the control center in a factory, or alternatively warn about smoke on a remote forest hill indicating that a dangerous fire is about to start. Other sensor nodes can be designed to detect the ground vibrations generated by the silent footsteps of a cat burglar and trigger an alarm.

Typically, the question that immediately following the actual detection of the events, is: *where*? Where are the abnormal vibrations detected, where is the fire, which house is about to be robbed? To answer this question, a sensor node needs to possess knowledge of its physical location in space. Location awareness is not only needed when reporting the geographical origin of events, but is critical in many other aspects of sensor networks as well. It can assist in routing in large scale ad-hoc networks[6] [7] or be used to study the coverage properties of the network. Furthermore, queries can now be extended to a group of nodes located in a specific area. We can envision another powerful use in a tactical scenario context, where a sensor network is designed to track the movements of targets inside its perimeter. Based on the knowledge of their own location, a group of nodes can discover the direction of the target movement and wake up other nodes (which are in sleep mode to conserve power) to continue the tracking task. Knowledge of location also drives context aware services[5]. In a 'smart kindergarten', localization techniques can keep track of the toys in an effort to monitor the children's progress. Other future uses may include navigation help for the blind, keeping track of equipment in hospitals, and many more. Although the above list is by no means complete, location awareness of a node is an essential requirement for many applications of sensor networks. The incorporation this location awareness in wireless sensor networks is far from a trivial task. Since the network can consist of a large number of nodes that are deployed in an ad-hoc fashion, manually programming each node with its geographic information is impossible. Furthermore, any dynamic technique must take into account the low-power constraints that govern every aspect of sensor networks. Indeed, unintrusive operation in their environment requires the node to be tetherless and to have small footprint. It therefore has to operate on small batteries. In most application scenarios, regular maintenance (and battery recharging) is virtually impossible, such that energy efficiency becomes a critical requirement for these nodes. Any technique for distributed location resolution therefore has to exhibit power awareness as well. This restriction is exactly what inhibits us from using the obvious solution of incorporating a Global Positioning System (GPS) receiver in every node. The objections against this option can be summarized as follows.

- As mentioned before, the power consumption of GPS is substantial. It directly affects the battery lifetime of the individual nodes and thus the overall behavior of the network.

• GPS requires line-of-sight signal reception from the GPS satellites and therefore does not work in the presence of dense vegetation, buildings or other obstacles.

• Also the cost factor of GPS can become an issue when large numbers of nodes are to be produced.

We therefore need another, more efficient alternative to provide dynamic location discovery in ad-hoc sensor networks, avoiding the drawbacks of GPS. This issue is exactly the one we will tackle in this paper.

Fig. 1. WINS Sensor Node from RSC

B. Our Work

We propose a new distributed technique that only requires a limited fraction of the nodes to know their exact location (either through GPS or manual configuration) and that nevertheless can attain network-wide fine-grain location awareness. Our technique, which we call AHLoS (Ad-Hoc Localization System), relieves the drawbacks of GPS as only a limited number of nodes suffer from the associated overhead. AHLoS enables other nodes to dynamically discover their own location, based on those few nodes that initially know their geographic position, essentially using them in the same way as the satellites in the GPS system. The main difference with GPS is that location information is gradually being distributed into the ad-hoc network as nodes compute their locations. This increases the points of reference that nodes can use to infer their locations. The location discovery of a node can be viewed as a two phase process. First, the distance between that node and those with a know position has to be estimated. Second, these distance estimates together with the known positions are combined to provide an approximate location of the unknown node.

The first phase, distance estimation, is known as $ranging$. Since almost all ranging techniques rely on signal propagation characteristics, they are susceptible to external factors such as interference, multipath effects and changes in temperature and humidity. These physical effects are difficult to predict and depend greatly the actual operation environment. It is therefore critical to characterize the behavior of different ranging alternatives experimentally in order to determine their usefulness in sensor networks. We have performed an elaborate comparison of two promising ranging techniques: one based on received RF signal strength and the other on ultrasound. Our experiments of distance discovery with RF signal strength are conducted on the the WINS wireless sensor nodes [13] (figure 1) developed by the Rockwell Science Center (RSC). To perform our evaluation of the ultrasound technique, we have designed and implemented a testbed of new ultrasound-equipped sensor nodes, called $Medusa$ (from Greek mythology - a monster with many heads) nodes (figure 2). As mentioned before, the propagation characteristics for both RF and ultrasound are dependent on the environment and our ranging technique has to be robust againt these variations. We therefore include in AHLoS the provision to dynamically estimate the propagation characteristics on the fly as they occur in the actual deployment environment.

The second phase in the location estimation is combining the range measurements into a new location estimate. We advocate a distributed algorithm, which we show to be more power efficient than a centralized solution. Furthermore, our approach is based on an iterative process, where increasingly more nodes are able to resolve their location. Our new technique, which we call **iterative multilateration** enables us to limit the number of original nodes with location information to an absolute minimum and increases the robustness in the presence of unavoidable ranging errors. We have tested our entire AHLoS location discovery scheme on a testbed comprised of the first generation of $Medusa$ nodes and we are able to demonstrate the ability to estimate node locations in an ad-hoc setting with a few centimeters accuracy. We also present the integration of AHLoS with an ad-hoc routing protocol and we currenlty use the results of our testbed to study the effects of error propagation as the network scales.

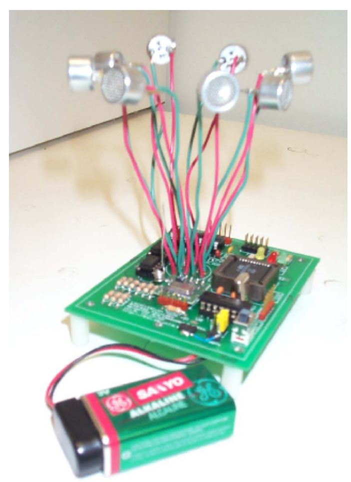

Fig. 2. $Medusa$ experimental node

C. Paper Organization

We start off with some background on localization and survey the related works in the next section. In section III we present our research methodology and examine the tradeoffs between the two $ranging$ solutions we have evaluated. In section IV, we present the **iterative multilateration** algorithm. The convergence of this algorithm depends on the network connectivity and the availability of beacon nodes, which is analyzed in section V. In section VI we describe our testbed setup, experimentation and power characterization of the $Medusa$ nodes. In section VII, we examine the tradeoffs between centralized and distributed approaches. Finally, section VIII outlines our plans for future work and concludes the paper.

II. Background and Related Work

A. Background

As we have mentioned in the introduction, any existing location discovery approach consists of two basic phases: (1) distance (or angle) estimation and (2) distance (or angle) combining. Several methods for estimating the distance between two nodes (first phase) exist. A detailed discussion of these methods can be found in [23].

- **Received Signal Strength Indicator (RSSI)** techniques measure the power of the signal at the receiver. Based on the known transmit power, the effective propagation loss can be calculated. Theoretical models allow us to translate this loss into a distance estimate. This method has been used mainly for RF signals.
- **Time based methods (ToA,TDoA)** record the time-of-arrival (ToA) or time-difference-of-arrival (TDoA). The propagation time can directly be translated in a distance, based on the known propagation speed. These methods can be applied to many different signals, such as RF, acoustic, infrared and ultrasound ones.
- **Angle -of -Arrival (AoA)** based systems strictly speaking do not measure distance. Instead they estimate the direction of the received signal and use simple geometric relationships to calculate node positions.

We evaluate both received signal strength and time based approaches for our localization technique (section III). Several alternatives also exist for the second phase of combining the distance measurements into actual node locations.

- The most basic and intuitive method is called hyperbolic trilateration. It locates a node by calculating the intersection of 3 circles (figure 3a).
- Triangulation is used when the direction of the node instead of the distance is estimated, as in AoA systems. The node positions are calculated in this case by using the trigonometry laws of sines and cosines (figure 3c).
- The third method is Maximum Likelihood (ML) estimation (figure 3b). It estimates the position of a node such that the differences between the measured distance and the distance from the estimated position to the known nodes, are minimized. This method is directly scalable to more nodes with known locations and also can incorporate weights to account for differences in distance estimate reliabilities. We have therefore chosen this technique as the basis of AHLoS.

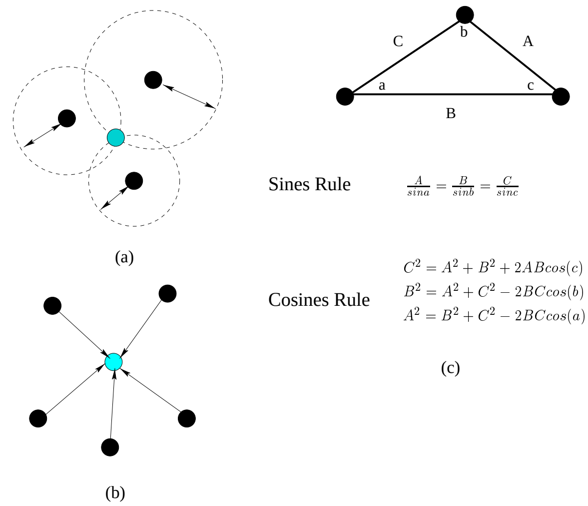

Fig. 3. Localization Basics a) Hyperbolic tri-lateration, b) ML Multilateration, c) Triangulation

B. Related Work

Most existing systems that perform localization are based on the existence of a fixed infrastructure. Since such infrastructure is absent in sensor networks, these techniques are not directly applicable here. The basic principles nevertheless can provide useful insights. We therefore first discuss some of the infrastructure-based systems. In the past few decades, a lot of systems have been based on RF signals. In the 1970s, these signals were used in the automatic vehicle location (AVL) system, which determines the position of police cars and military ground transportation. This system consists of a set of stationary base stations as observation points and uses ToA and TDoA techniques to generate distance estimates. The vehicle position is derived through multilaterations, using Taylor Series Expansion to linearize the optimization problem [8][9]. Similar approaches were also found in military applications for determining the position of airplanes. In the 1990s, RF based localization has received attention in the cellular world as well. At that point, the U.S Federal Communications Commission (FCC) required that all wireless service providers give location information to the Emergency 911 services. Cellular base stations are locate mobile telephone users within a cell [10][11]. Distance estimates are generated with TDoA, since this technique has the advantage that the handset only needs to reflect the signal back to the basestation and therefore does not require additional hardware. Location information is calculated using least square methods. For this system FCC requires a 125 meter RMS accuracy in 67 percent of the time. In 1993, the well-known GPS [**?**] system was deployed, which is based on the NAVSTAR satellites constellation (24 satellites). A similar system, called LORAN [32], uses ground based beacons. An RF based system to track the location of users within a building is the RADAR system [1]. It uses RF signal strength measurements from three fixed base stations. These measurements are sent to a central server to translate them into distances. This translation is based on signal strength maps that were generated in advance for a number of sites inside the building. Thanks to this process, the effects of shadowing can be counteracted, at the cost of a considerable preplanning effort.

The Cricket location support system [5] is also designed for indoor localization. It provides support for context aware applications and is low cost. Unlike all of the systems discussed so far, it uses ultrasound instead of RF signals. Fixed beacons inside the building distribute geographic information to the listener nodes. Cricket can achieve a granularity of 4 by 4 feet. A finer granularity is obtained by the active badge [25] system, which uses infrared. The next development in this area on indoor localization is BAT [33] [34]. A BAT node carries an ultrasound transmitter of which the signals are picked up by an array of receivers mounted on the ceiling. The location of a BAT can be calculated via multilateration up to an accuracy of a few centimeters. An RF basestation coordinates the ultrasound transmissions such that interference from nearby transmitters is avoided. This system relies heavily on a centralized infrastructure.

Fewer localization systems for ad-hoc networks exist, compared to all the infrastructure based ones that were discussed so far. A first approach is presented in [24], where location of a node is given as a centroid. This centroid is generated by counting beacon signals that are sent by a set of beacons preppositioned in a mesh pattern. A second approach is taken in the Picoradio project at UC Berkeley. It provides a geolocation scheme for an indoor environment [12], based on RF received signal strength measurements and pre-calculated signal strength maps.

Our system, AHLoS, also belong to the ad-hoc class. It uses RF and ultrasound transmissions similar to the Cricket and Bat Systems, but with some key differences. The operation does not rely on an infrastructure setting. Instead it is a fully ad-hoc system with distributed localization algorithms running at every node. From a power awareness point of view, it also has the important property that all nodes play an equal role in the location discovery process. Our technique provides fine-grained localization with an accuracy of a few centimeters, similar to the BAT system but does not require infrastructure support. Unlike all the systems discussed so far, we provision for a dynamic on-line estimation of the ultrasound propagation characteristics. This renders our approach extremely robust even in the presence changing environments.

For our system, we have considered two options of measuring distances. The first one is based on RF received signal strength (RSSI), and the second one looks at the arrival time of ultrasound signals. In the next subsection, we present a detailed study of the propagation characteristics of both options. The results were obtained via actual field experiments on the WINS (for RF RSSI) and $Medusa$ (for ultrasound TDoA) nodes.

III. Research Methodology

The fundamental building block of most localization techniques relies on the ability to measure distances to a known reference point. In an ad-hoc setting, there is no fixed point of reference that one can use for $ranging$. Furthermore, for purposes of robustness and power conservation, one would like all the nodes in the network to play an equal role in the location discovery process. As such, for our purposes we define $ranging$ as the ability of adjacent nodes, (that are within communication and sensing range with each other) to estimate their separation distance. As a first step, we motivate our approach by first characterizing the $ranging$ capabilities of our two target systems: RF signal strength measurement using the WINS nodes and ultrasound measurements using the $Medusa$ nodes. We evaluate the tradeoffs of the two solution approaches in terms of their $ranging$ accuracy.

A. Ranging Characterization

A.1 Received Signal Strength

The signal strength method uses the relationship of RF signal attenuation as a function of distance. From this relationship a mathematical propagation model can be derived . From detailed studies of the RF signal propagation characteristics[19], it is well known that the propagation characteristics of radio signals can vary with changes in the surrounding environment (weather changes, urban / rural and indoor / outdoor settings). To evaluate signal strength measurements we conducted some experiments with the target system of interest the WINS sensor nodes [13]. WINS have a 200MHz StrongARM 1100 processor, 1MB Flash, 128KB RAM and the Hummingbird digital cordless telephony (DECT) radio chipset that can transmit at 15 distinct power levels ranging from -9.3 to 15.6 dBm (0.12 to 36.31 mW). The WINS nodes also have an omni-directional antenna hence the radio signal is uniformly transmitted with the same power in all directions around the node. These nodes can carry a variety of sensors such as seismic and acoustic sensors. As part of the radio architecture the WINS nodes provide a pair of RSSI (Received Signal Strength Indicators) resisters. RSSI registers are a standard feature in many wireless network cards [26]. Using these registers we conducted a set of measurements in order to derive an appropriate model for ranging.

We also attempted measurements in several different settings (inside our lab, in the parking lot and between buildings). Unfortunately, in these settings we could not obtain a consistent model of the signal attenuation as a function of distance. This is mainly attributed to the multipath fading and shadowing effects. Another inconsistency we noted with RSSI is related to the height of the radio antenna. At ground level, the radio range at the maximum transmit power level is around 30m while when we placed one of the nodes at a height of 1.5m the usable transmission range increased to around 100m. Because of these inconsistencies, we were only able to derive a model for an idealized setting; in a football field with all the nodes positioned at ground level. For this setup we developed a model based on the RSSI register readings at different transmission power levels and different node separations.

A model (equation 1)is derived by obtaining a least square fit for each power level. P_{RSSI} is the RSSI register reading and r is the distance between two nodes. Parameters X and n are constants that can be derived as functions of distance r for each power level. Averaged measurements and the corresponding derived models are shown in figures 4 and 5. Table I gives the X and n parameters for each case.

$$P_{RSSI} = \frac{X}{r^n} \tag{1}$$

Assuming that all the nodes are placed on a flat plane, signal strength ranging can provide a distance estimate with an accuracy of a few meters. This experiment has shown that the use of radio signal strength can be very unpredictable. A problem with the RSSI approach is that radios in sensor nodes are low cost ones without precise well-calibrated components, such as the DECT radios in Rockwell's nodes or the emerging Bluetooth radios. As a result it is not unusual for different nodes

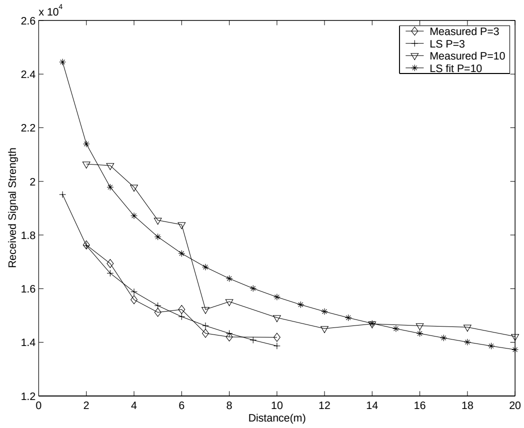

Fig. 4. Radio Signal Strength Radio Characterization using WINS nodes(Levels 3,10)

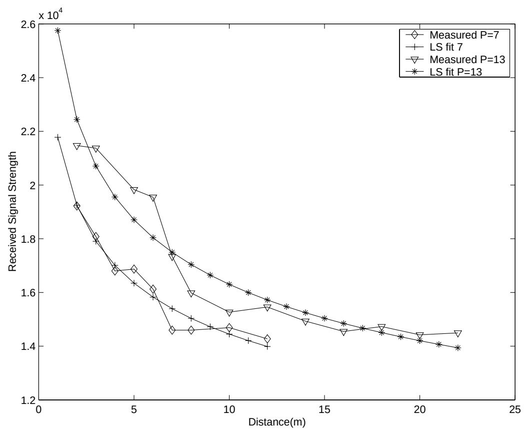

Fig. 5. Radio Signal Strength Radio Characterization using WINS nodes(levels 7,13

to exhibit significant variation in actual transmit power for the same transmit power level setting, or in the RSSI measured for the same actual received signal strength. Differences of several dBs are often seen. While these variations are acceptable for using transmit power adaptation and RSSI measurements for link layer protocols, they create problems for localization. A potential solution would be to calibrate each node against a reference node prior to deployment, and store gain factors in non-volatile storage so that the run-time RSSI measurements may be normalized to a common scale.

From our experience we concluded that albeit the signal strength method may be a suitable candidate for determining node proximity (as in [24]), it would be very hard to use for fine grained localization especially in an ad-hoc setup. In an indoor or urban setting, one can use this received signal strength method by constructing signal strength maps as in [1] and [12]. This approach however, does require some preplanning so it is not really ad-hoc. To overcome the problems caused by the signal strength amplitude variations of RF reception, we adopt a time difference of arrival(TDoA) approach that makes use of the time difference between simultaneously transmitted radio and ultrasound pulses to measure distances. We evaluate this approach using the $Medusa$ nodes we have build. Next, we describe the $Medusa$ node architecture.

TABLE I

RSSI RANGING MODEL PARAMETERS FOR WINS NODES

Power Level	dBm	mW	X	n
3	-5.2	0.302	19509	0.148328
7	2.5	1.78	21778.338	0.178186
10	10	10	24449.78	0.1926
13	14.4	27.54	25753.63	0.198641

B. Medusa Node Architecture

The *Medusa* node design 2 is based on the AVR 8535 processor [14] which carries 8KB of flash memory, 512 bytes SRAM and 512 bytes of EEPROM memory. The radio we use is a DR3000 module from RF Monolithics[15]. The ultrasound circuitry consists of five pairs of 40KHz transceivers arranged in a pentagonal pattern at the center of the board. Each ultrasound transceiver is supported by a pair of solid core wires at an approximate height of 15 cm above the printed circuit board. We found this very convenient setup for experimentation since it allows the transceivers to be rotated in different directions. The first generation board is 3” x 4” and it is based on full size components and it is powered by a 9V battery. We expect that future generations will have surface mount components and a smaller footprint. The current version of the board is intended as a research and experimentation platform with several test points and full size components, hence the larger footprint. The $Medusa$ firmware is based on an event driven firmware implementation [16]. It includes a variable size framing scheme, 4-6 bit encoding [17] and 12 bit CRC. The code for $ranging$ is integrated in the ad-hoc routing protocol described in section VI-A.

B.1 TDoA using RF and Ultrasound

The $ranging$ experiment we have performed with the $Medusa$ nodes uses TDoA at the receiver of simultaneously transmitted radio and ultrasound signals, figure 6. Our choice is motivated by the fact that time difference methods are more tolerant to changing environments than amplitude based methods since the received amplitude can be affected by many unpredictable external factors.

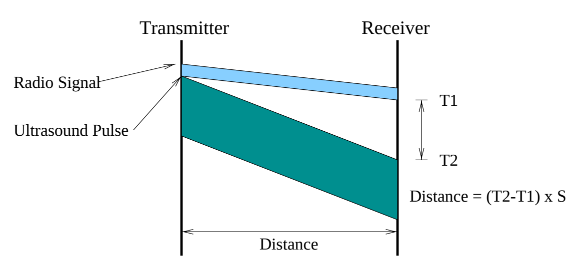

Fig. 6. Distance measurement using ultrasound and radio signals

The ultrasound range on the $Medusa$ nodes is about 3 meters (approximately 11-12 feet). The ultrasound pulses can be detected at larger distances (up to 12meters), but cannot provide an accurate distance measurement due to the loss of some of pulses. We found this to be a convenient range for performing multihop experiments in our lab but longer ranges are also possible. The Polaroid 6500 ultrasonic ranging module [18] for example has a range of more than 10 meters. Other systems have even greater ranges but these impose higher cost and power premiums. A detailed description of the $Medusa$ architecture is provided in section IV. The time between the radio and ultrasound transmissions is measured using a timer routine running on the microcontroller. By exploiting the linear relationship of sound propagation as a function of distance, the time difference between the reception of simultaneously transmitted radio an ultrasound signals provide a good approximation of range. Using these relationships we characterize TDoA ranging of the $Medusa$ nodes by placing them on the floor at different separations. The results from this characterization are shown in figure 7. The x axis represents distance in centimeters and the y axis represent the microcontroller timer counter value.

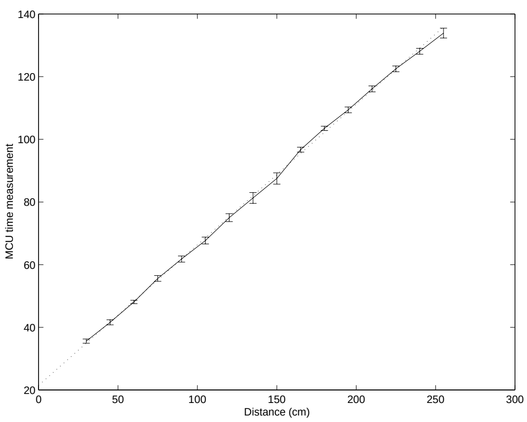

Fig. 7. Ultrasound Ranging Characterization

The speed of sound is characterized in terms of the microcontroller timer ticks. To estimate the speed to sound as a function of microcontroller time, we perform a best line fit using linear regression (equation 2). s is the speed of sound in timer ticks, d is the estimated distance between 2 nodes and k is a constant. For this model $s = 0.4485$ and $k = 21.485831$.

$$t = sd + k \tag{2}$$

This $ranging$ system can provide an accuracy of 2 centimeters for node separations under 3 meters. Another convenient property of ultrasound is that most of the multipath effects can be detected and filtered out. Since the variance between multiple measurements of the same separation distance is very small, measurement inconsistencies due to multipath effects can be easily detected and filtered out. One drawback of this $ranging$ system that the ultrasound signal can be stopped by obstacles. In the next subsection we proceed with a comparison of between ultrasound and signal strength ranging.

C. Signal Strength vs. Ultrasound Ranging

On comparing the 2 ranging alternatives, we found that ultrasound is more reliable than signal strength. While signal strength is greatly affected by amplitude variations of the received signal, ultrasound $ranging$ only depends on the time difference, a much more robust metric. In both cases, the signal propagation characteristics may change with variations in the surrounding environment. To minimize these effects, our proposed solution dynamically estimates the signal propagation characteristics every time sufficient location information is available. This ensures that AHLoS will operate in many diverse environments without prior calibration. If the sensors are deployed over a wide area, the signal propagation characteristics may be vary across the region of interest. By calculating the propagation characteristics locally we can achieve better accuracy in the sensor location estimates. Table II summarizes the comparison between signal strength and ultrasound ranging. Based on our based on the outcome of our ranging characterizations, we select TDoA as our primary ranging method. We therefore continue our discussion of localization using the ultrasound TDoA method and the $Medusa$ nodes as our ranging platform. One possible solution that we are considering for our future work is to combine signal strength and ultrasound methods. Since the signal strength method has the same effective range as the radio communication range, it can be used to provide a location indication in places where the network is sparse. The ultrasound approach will provide fine grained localization in denser parts of the networks. For this configuration, we plan to have the $Medusa$ boards act as *location coprocessors* for the WINS nodes.

IV. Localization Algorithms

Given a $ranging$ technology that estimates node separation we can now describe the **iterative multilateration** algorithm. The algorithm assumes a network of nodes where a small percentage of the nodes are aware of their positions either through manual configuration or using GPS. We define these nodes as $beacon$ nodes. These nodes broadcast their location information to their neighbors and when sufficient information becomes available they can also estimate their locations. The rest of the nodes are not aware of their locations and they are defined as $unknown$. The main objective of the **iterative multilateration** algorithm is to enable as many of the unknown nodes to estimate their locations. An $unknown$ node can estimate its position by one of the multilateration processes described below. Once a node estimates its position it can become a $beacon$ and broadcast its location information to the rest of the nodes, thus enabling more and more nodes to calculate their locations. The localization process is complete when all the $unknown$ nodes that meet a certain set of criteria can compute their locations. The location discovery has to meet following challenges:

- The $ranging$ estimates and the position estimates at the beacon nodes will contain errors. Besides considering these errors when calculating a node's position, one must also ensure that the errors will not propagate throughout the network.
- In some cases , there may not be enough beacon nodes to perform a successful multilateration. In this case we apply a more complex *collaborative multilateration* procedure.
- It must operate in a distributed manner that is fault tolerant, and can handle local variations in the propagation conditions.

TABLE II
A COMPARISON OF RSSI AND ULTRASOUND RANGING

Property	**RSSI**	**Ultrasound**
Range	same as radio communication range	3 meters (up to a few 10s of meters)
Accuracy	O(m), 2-4m for WINS	O(cm), 2cm for Medusa
Measurement Reliability	hard to predict, multipath and shadowing	multipath mostly predictable,time is a more robust metric
Hardware Requirements	RF signal stregth must be available to CPU	ultrasound transducers and amplifier circuitry
Additional Power Requirements	none	tx and rx signal amplification
Challenges	large variances in RSSI readings, multipath, shadowing, fading effects	interference, obstacles, multipath

In the following subsections we first define the two primary components and later on using these components we define iterative multilateration.

A. Atomic Multilateration

Consider the case of a single multilateration in figure 8a. The error distance for the observation of the ith beacon node can be expressed as the difference between the $ranging$ and the Euclidian distance between a pair of adjacent nodes (equation 3).

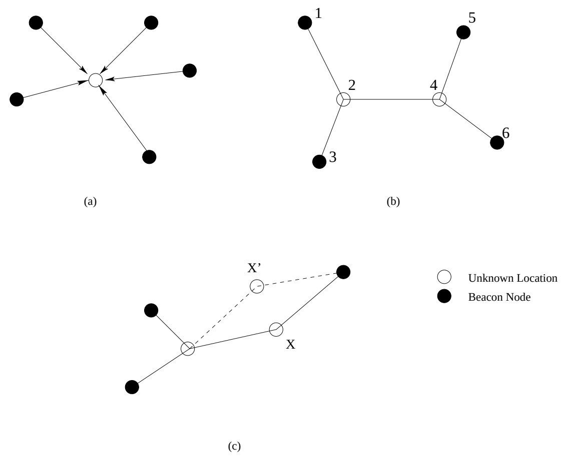

Fig. 8. Multilateration examples

$$f_i(x_0, y_0, s) = st_{i0} - \sqrt{(x_i - x_0) - (y_i - y_0)} \qquad (3)$$

Given that an adequate number of beacon nodes are available, a Maximum Likelihood estimate of the node's position can be obtained by taking the minimum mean square estimate (MMSE) of a system of $f(i)$ equations (equation 4). Term α represents the weight applied to each $f(i)$. In this basic case we solve for all $\alpha_i = 1$.

$$F(x_0, y_0, s) = \sum_{i=1}^{2N} \alpha_i^2 f(i)^2 \qquad (4)$$

This overdetermined system of equations can be linearized so that a matrix solution can be used. by expanding equation (1) and rearranging terms we get

$$-x_i^2 - y_i^2 = (x_0^2 + y_0^2) + x_0(-2x_i) + y_0(-2y_i) - s^2 t_{i0}^2 \qquad (5)$$

for k such equations we can eliminate the $(x_0^2 + y_0^2)$ term by subtracting the last equation from the rest.

$$\begin{aligned} -x_i^2 - y_i^2 + x_k^2 + y_k^2 &= 2x_o(x_k - x_i) \\ &+ 2y_0(2y_k - y_i) + s^2({t_{ik}}^2 - {t_{i0}}^2) \end{aligned} \qquad (6)$$

this system of equations has the form $y = bX$ and can be solved using the matrix solution for MMSE [28] given by $b = (X^T X)^{-1} X^T y$

where

$$X = \begin{bmatrix} 2(x_k - x_1) & 2(y_k - y_1) & {t_{k0}}^2 - {t_{k1}}^2 \\ 2(x_k - x_2) & 2(y_k - y_2) & {t_{k0}}^2 - {t_{k2}}^2 \\ \vdots & \vdots & \vdots \\ 2(x_k - x_{k-1}) & 2(y_k - y_{k-1}) & {t_{k0}}^2 - {t_{k(k-1)}}^2) \end{bmatrix}$$

$$Y = \begin{bmatrix} -x_1^2 - y_1^2 + x_k^2 + y_k^2 \\ -x_2^2 - y_2^2 + x_k^2 + y_k^2 \\ \vdots \\ x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2 \end{bmatrix}$$

and

$$b = \begin{bmatrix} x_0 \\ y_0 \\ s^2 \end{bmatrix}$$

We note that here we can have 2 alternative formulations. By moving the st_{i0} term to the left hand side in equation 5 we assume that the ultrasound propagation parameters are known. This solution requires only 3 beacon nodes. The solution provided here is for the case where 4 or more $beacon$ nodes are available. In this case the ultrasound propagation characteristics can be estimated dynamically.

Definition 1: For the atomic multilateration to take place it is necessary and sufficient that the node is within 1 hop distance from at least 3 $beacon$ nodes. To estimate the speed to of sound locally, 4 or more beacons are required.

B. Collaborative Multilateration

Since we consider an ad-hoc deployment, often the conditions for atomic multilateration may not be met; i.e an $unknown$ node may never have 3 neighboring $beacon$ nodes therefore it will not be able to estimate its position using *atomic multilateration*. If this occurs, a node may attempt to estimate its position by making use of location information over multiple hops. If sufficient information is available to form an overconstrained system of equations with a unique solution set, a node can estimate its position by solving a set of simultaneous quatratic equations. This calculation will yield a position estimate for two or more unknown nodes. We refer to this process as *collaborative multilateration*. Figure 8b illustrates the most basic case for which *collaborative multilateration* can be used. Nodes 2 and 4 are $unknown$ nodes, while nodes 1,3,5,6 are $beacon$ nodes. Since both 2 and 4 have degree $d = 3$ and all the other nodes are $beacons$ an overdetermined system of equations with a unique solution set can be formed. More formally, collaborative multilateration can be stated as follows: Consider the ad-hoc network to be a connected undirected graph $G = (N, E)$ consisting of $|N| = n$ nodes and a set E of $n - 1$ or more edges. The $beacon$ nodes are denoted by a set B where $B \in G$ and the set of $unknown$ nodes is denoted by U where $U \in G$. Our goal is to solve for

$x_u, y_u \; \forall u \in U$ by minimizing

$$f(x_u, y_u) = D_{ij} - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \qquad (7)$$

for all participating node pairs i, j and all unknown nodes U Subject to:

- $x_b, y_b \; \forall b \in B$,
- each participating node having degree $d \geq 3$ with at least 3 neighbors that are either $beacons$ or participating nodes. In figure 8b if collaborative multilateration starts at node 2, node 2 must have at least 3 participating neighbors. Nodes 1,3 are beacons therefore they are participating. Node 4 is unknown but has 2 beacons 5 and 6. Node 4 is also connected to node 2 thus making both of them participating nodes.

In this formulation, the nodes participating in *collaborative multilateration* is a subgraph of G, for which an equation of the form of 7 can be formed for each edge E in the participating node set. Again, in figure 8b, we have 5 edges thus a set of 5 equations can be obtained. In some cases, a system of n equations and n unknowns may have more than one solution. In the network in figure 8c for example, set of 4 non-linear equations and 4 unknowns can be obtained. We can easily observe however, that node X can have 2 possible positions that would satisfy this system therefore the solution is not unique and node X is not a *participating* node. These systems of non-linear equations can be solved with optimization methods such as gradient descend [30] and simulated annealing [31].

Definition 2: A node can participate in a collaborative multilateration if it is a $beacon$ or it has at least 3 neighbors which are either $beacon$ nodes or they have $unknown$ locations but they have at least 3 participating neighbors.

The algorithm in figure 9 illustrates how a node can determine whether it can initiate *collaborative multilateration*. The parameter *node* denotes the node id from where the search for a collaborative multilateration begins. *isInitiator* is a boolean variable that is set to *true* if the node was the initiator of the process and *false* otherwise. This is used to set the *limit* flag that drives the recursion. This algorithm is used in the next subsection to study the contribution of the collaborative multilateration algorithm in the iterative multilateration process (figure 11).

```
boolean isCollaborative (node, isInitiator)
  if isInitiator==true limit ← 3
  else limit ← 3
  count ← beaconCount(node)
  if count ≥ limit return true
  for each unknown neighbor i not previously visited
    if isCollaborative (i, node, false) count++
    if count == limit return true
  return false
```

Fig. 9. Algorithm for checking the feasibility for collaborative multilateration

C. Iterative Multilateration

The **iterative multilateration** algorithm uses *atomic* and *collaborative multilateration* as building blocks to estimate node locations over an ad-hoc network. This algorithm is fully distributed and can run on each individual node in the network. Alternatively, the **iterative multilateration** algorithm can run at a singe central node or a set of cluster head nodes, if the network is cluster based. In any scenario, the computation takes place in a distributed manner. When sufficient location information becomes available at a node, a position estimate can be computed. Once a location estimated is computed, the node becomes a beacon. Initially, each node will attempt to estimate its position using *atomic multilateration*. If the conditions for *atomic multilateration* are not met for a certain time threshold, a node will try to determine if enough information is available in its immediate neighborhood for *collaborative multilateration*. If the node can obtain all the information needed for *collaborative multilateration*, it will either calculate the node position estimates locally, if enough computation power is available, it it will send the information to a remote node that can perform the computation and send back the results. Since *collaborative multilateration* is computationally very expensive compared to *atomic multilateration*, the **iterative multilateration** tries to minimize its use. Figure 10 provides a pseudocode listing for iterative multilateration. Since at a central location all the information is known, always start a multilateration at the $unknown$ node with the maximum number of beacons to obtain better accuracy and faster convergence.

Collaborative multilateration can help in situations where the percentage of beacons is low. This effect is shown in figure 11. This scenario considers a sensor field of 100 by 100 and sensing range of 10 and two network sizes of 200 and 300 nodes. As shown in the figure, at small percentage of $beacons$, the percentage of node locations that can be resolved is substantially increased with collaborative multilateration. This result

```
boolean iterativeMultilateration (G)
  (MaxBeaconNode, BeaconCount) ← unknown
    node with most beacons
  while BeaconCount ≥ 3
    setBeacon (MaxBeaconNode)
   (MaxBeaconNode, BeaconCount) ← unknown
     node with most beacons
    while isCollaborative (MaxBeaconNode, -1, true)
      set all nodes in collaborative set as beacons
     (MaxBeaconNode, BeaconCount) ← unknown
       node with most beacons
      while BeaconCount ≥ 3
        setBeacon (MaxBeaconNode)
         (MaxBeaconNode, BeaconCount) ← unknown
           node with most beacons
```

Fig. 10. Iterative Multilateration Algorithm as it runs from a centralized node

also shows how network density is related to the localization process. In the 300 node network, more node locations can be estimated than in the 200 node network with the same percentage of beacons. This is due to the higher degree of connectivity. The effects of node and beacon placement on the localization process is studied in more detail in section V

V. NODE AND BEACON PLACEMENT

The success of the location discovery algorithm depends on network connectivity and beacon placement. In this section, we first conduct a probabilistic analysis of to determine how the connectivity requirements can be met is nodes are to be uniformly deployed in a field. Based on these results, in the second part of this section we carry out a statistical analysis to get an indication on the percentage of beacons that need to be deployed. When considering node deployment, the main metric of interest is the probability with which any node in the network has a degree of 3 or more, assuming that sensor nodes are uniformly distributed over the area of the sensor field. In a network of N nodes, the probability $P(d)$ of a node having degree d is given by the binomial distribution in equation 8 and the probability P_R being in transmission range.

$$P(d) = P_R^d . (1 - P_R)^{N-d-1} . \binom{N-1}{d} \tag{8}$$

$$P_R = \frac{\pi R^2}{L^2} \tag{9}$$

For large values of N tending to infinity, the above binomial distribution converges to a Poisson distribution. When taking into account that $\lambda = N.P_R$ we get equation 10, the probability of a node have degree of three or more can be calculated. In terms of λ, an indication of the number of nodes per unit area to achieve a certain densities can be calculated. Table III shows the results for different values of λ. The fourth column in the table shows the number of nodes (with range 10) that are needed to achieve the specified value of λ using a uniform deployment in a field of 100 by 100. The probability of a node having a degree of $d \geq n$ can be calculated as in equation 11.

$$P(d) = \frac{\lambda^d}{d!} . e^{-\lambda} \tag{10}$$

$$P(d \geq n) = 1 - \sum_{i=0}^{n-1} P(i) \tag{11}$$

TABLE III
PROBABILITY OF NODE DEGREE FOR DIFFERENT λ VALUES

λ	P($d \geq 3$)	P($d \geq 4$)	nodes/10,000m^2
2	0.323324	0.142877	39
4	0.761897	0.56653	78
6	0.938031	0.848796	117
8	0.986246	0.95762	157
10	0.997231	0.989664	196
12	0.999478	0.997708	235
14	0.999906	0.999526	274
16	0.999984	0.999907	314
18	0.999997	0.999982	353
20	1	0.999997	392

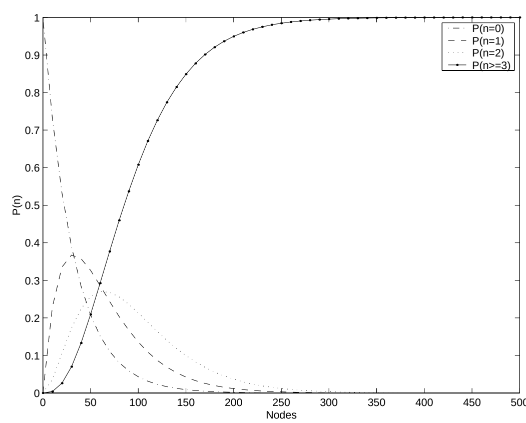

Fig. 12. Connectivity result for a 100 x 100 field and sensor range 10

Figure 12 shows the connectivity results for an area measuring 100 by 100 with node sensing range of 10. The probabilities for nodes having 0,1 and 2 neighbors are also shown in this figure.

In addition to node connectivity, the next important aspect is to determine the percentage beacons required to assist the convergence of the localization algorithm. The feasibility of localization is determined this by statistical analysis. In a sensor field with dimensions 100 x 100, sensors with sensing range of 10 are deployed. For this setup, we report the percentage of nodes that estimate their locations while varying the percentage of nodes and beacons. The results shown in figure 13 show are averages

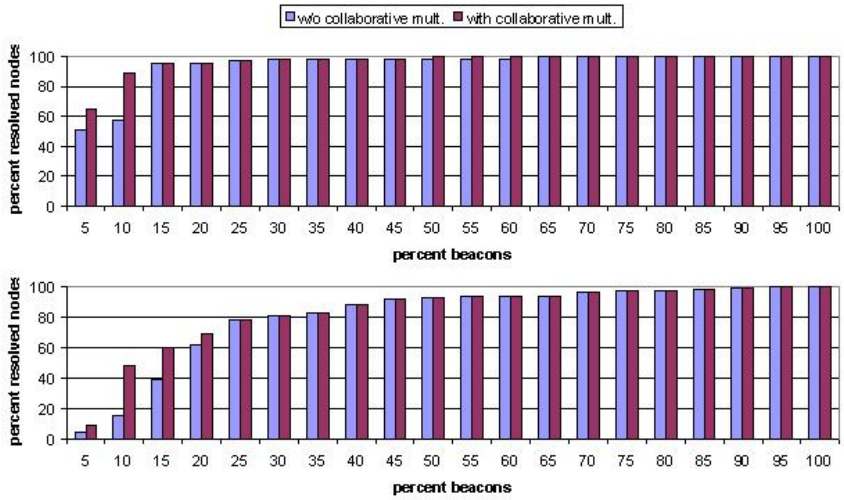

Fig. 11. Effect of collaborative multilateration top, 300 nodes, bottom 200 nodes

over 100 simulations. The figure shows that the percentage of beacons required to complete the iterative multilateration process decreases as with increasing beacon densities. Also as the network density increases, the transitions in the required number of beacons become much sharper since the addition of a few more beacon nodes reinforces the progress of the iterative multilateration algorithm.

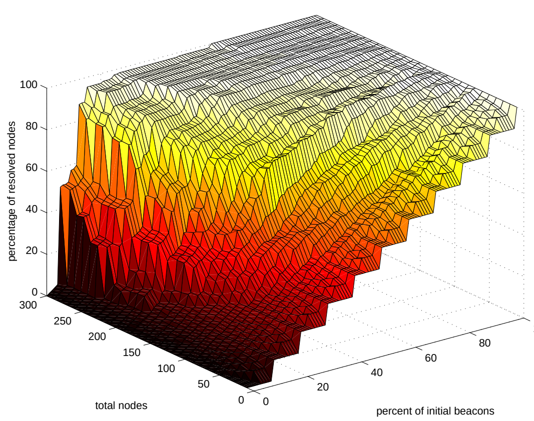

Fig. 13. Beacon Requirements for different node densities

VI. EXPERIMENTATION

Our experimental testbed consists of 9 $Medusa$ nodes 2 and one Pentium II 300MHz PC. One $Medusa$ node is attached to the PC and acts as a gateway for the rest of the nodes. The PC runs the localization algorithms presented in IV. Some of the nodes are programmed with locations so they can act as *beacons* while the rest of the nodes are *unknowns*. The nodes perform the ranging and forward all the location and ranging information to the PC. On the PC the localization algorithms will calculate the location estimates and display them on our sensor visualization tool(figures 14 and 15).

The node positions on the sensor visualization tool are updated at 5 second intervals. Figures figures 14 and 15 show some snapshots of node locations. The beacons are shown as black dots, the unknown nodes are white circles and the node position estimates are shown as gray dots. In all of our experiments all the node position estimates for each unknown node always fall within the 3" x 4" surface area of the $Medusa$ boards. In the next subsection we describe how the node location information is transmitted to the PC by using a multi-hop routing protocol.

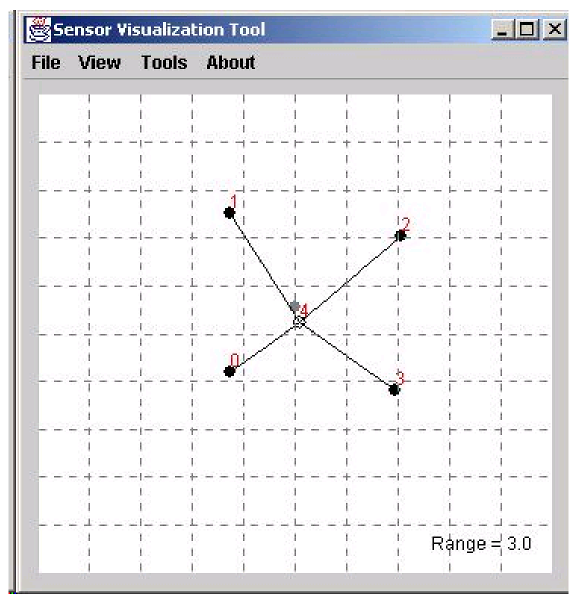

Fig. 14. 5 node schenario

A. Location Information Dissemination and Routing

To route messages to the base station, we use a lightweight version of the DSDV routing algorithm, DSDVlite. Instead of maintaining a routing table with the next hop information to every node, DSDVlite only maintains a short routing table that contains next hop information for the shortest route to the base station. Furthermore, this algorithm drives the localization pro-

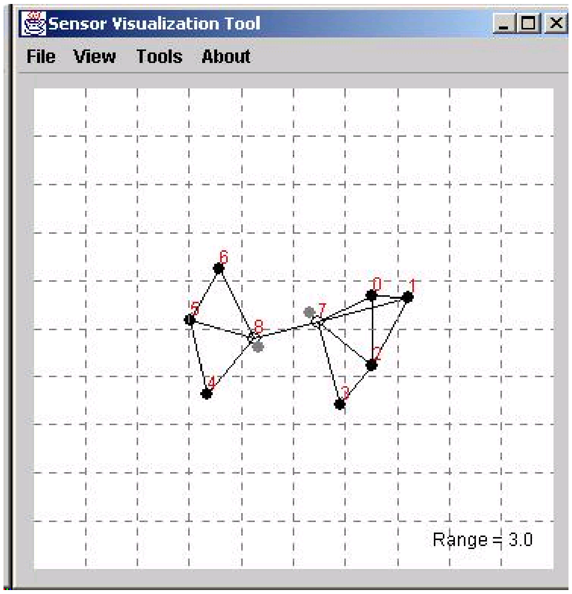

Fig. 15. 9 node scenario

cess by carrying the location information of the sender, and by ensuring that the ultrasound beacon signals are not misinterpreted at the receiver. The ultrasound beacon signal transmission begins right after the transmission of the start symbol for each routing packet. After this, the transmission of data and ultrasound signals proceed simultaneously. By ensuring that the duration of the data transmission is longer than the ultrasound transmission, the receiver can differentiate between erroneous ultrasound transmissions from other nodes. If the data packet is not correctly received because of a collision with another transmission, then the ultrasound time measurement is also discarded.

B. Power Characterization

In the previous subsection we verified the correct operation of our localization system. Our experimental setup will provide a reasonable solution for a small network but as the network scales, the traffic to the central node will increase substantially. Before we can evaluate the tradeoffs between estimating locations at the nodes and estimating locations at a central place we first characterize power consumption of the $Medusa$ nodes. Using an HP 1660 Logic Analyzer a bench power supply and a high precision resistor we characterized the RFM radio and the AVR microcontroller on the $Medusa$ nodes.

$$I_{sensor} = \frac{V_{test}}{R_{test}}$$

$$V_{Sensor} = V_{supply} - V_{test}$$

$$Power = I_{sensor} \times V_{sensor}$$

$$Energy = Power \times time$$

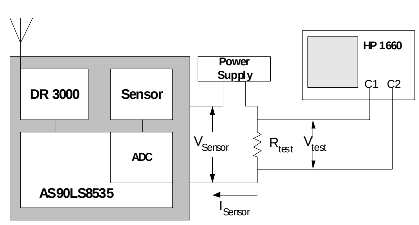

Fig. 16. a) Power and Energy Relationships, b) Measurement Setup

The measurement setup and power/energy relationships are shown in figure 16. The power consumption for different modes of the AVR microcontroller are shown in table IV. The power consumption for the different modes of the RFM radio are shown in figure 17 and table V

TABLE IV
AVR 8535 POWER CHARACTERIZATION

AVR Mode	Current	Power
Active	2.9mA	8.7mW
Sleep	1.9mA	5.9mW
Power Down	1μA	3μW

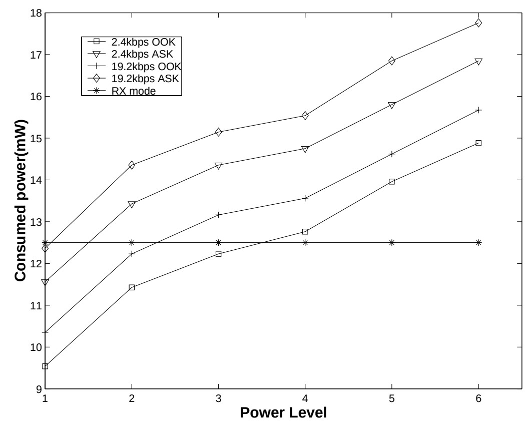

Fig. 17. RFM Radio Power Characterization

With the power characterization in hand we can now evaluate the tradeoffs between a centralized and distributed implementations of our localization protocols.

VII. TRADEOFFS BETWEEN CENTRALIZED AND DISTRIBUTED SCHEMES

One important aspect that needs to be determined is whether the location estimation calculation should be done at each node (distributed) manner or at a central node(centralized). We examine this problem in the context of the AHLoS system where each node needs to be aware of its location.

The calculation at a central node has several drawbacks. First, to forward the location information to a central node a route to the central node must be known. This implies the use of a routing protocol other than location based routing and also incurs some additional cost due to communication that also relies on the efficiency of the existing routing and media access control protocols. Second, it creates a time synchronization problem. Whenever there is a change in the network topology the node's knowledge of location will not instantaneously updated. To keep track of events correctly the central node will need to cache node locations to ensure consistency of event reports in space and time. Third, the placement of the central node implies some preplanning to ensure that the node is easily accessible by other nodes. Also, because of the large volume of traffic to and from the central node, the battery lifetime of the nodes around the central node will be seriously impacted. Fourth, the robustness of the system suffers. If the routes to the central node are

TABLE V
RFM POWER CHARACTERIZATION

Mode	Power Level	OOK Modulation				ASK Modulation			
		2.4Kbps		19.2Kbps		2.4Kbps		19.2Kbps	
		mA	mW	mA	mW	mA	mW	mA	mW
Tx	0.7368	4.95	14.88	5.22	15.67	5.63	16.85	5.95	17.76
Tx	0.5506	4.63	13.96	4.86	14.62	5.27	15.80	5.63	16.85
Tx	0.3972	4.22	12.76	4.49	13.56	4.90	14.75	5.18	15.54
Tx	0.3307	4.04	12.23	4.36	13.16	4.77	14.35	5.04	15.15
Tx	0.2396	3.77	11.43	4.04	12.23	4.45	13.43	4.77	14.35
Tx	0.0979	3.13	9.54	3.40	10.35	3.81	11.56	4.08	12.36
Rx	-	4.13	12.50	4.13	12.50	4.13	12.50	4.13	12.50
Idle	-	4.08	12.36	4.08	12.36	4.08	12.36	4.08	12.36
Sleep	-	0.005	0.016	0.005	0.016	0.005	0.016	0.005	0.016

broken, the nodes will not be able to communicate their location information to the central node and vice versa. Finally, since all the raw data is required at the central node, the network cannot aggregate the data to conserve power. Overall, a centralized implementation will not only reduce the network lifetime but it will also increase its complexity and compromise its robustness. On the other hand, if location estimation takes place at each node in a distributed manner the above problems can be alleviated. Topology changes will be handled locally and the location estimate at each node can be updated at minimal cost. In addition, the network can operate totally on location based routing so the implementation complexity will be reduced. Also since each node is responsible for determining its location, this is more tolerant to node failures and node mobility.

To further evaluate the tradeoff between the centralized and distributed setups we simulated a typical sensor network setup. In our scenario the central node is placed at the center of a square field. We measure the total number of bytes transmitted by all the nodes for both distributed and centralized networks. We vary the network size by we keep the network density constant by using a value of $\lambda = 6$ or 117 nodes for every 10,000m^2 (from table III). The simulation setup considers the same packet structures as the implementation on the medusa nodes. For the centralized system each node forwards the range measurements between all its neighbors. If the node is $beacon$ it will also forward its location information (this is 96 bits long to match the GPS readings). Once the location is computed, the central node will send each node its computed location estimate. In the distributed setup each node transmits a short beacon signal (radio and ultrasound pulse) followed by the senders location if this is available. In both cases, the simulation runs for one full cycle of the localization process. The results of these simulations are shown in figures 18 and 19.

Figure 18 uses 10% beacons and figure 19 uses 20% beacons. The results show that in the distributed setup creates six to 10 times less traffic than the centralized setup. Another interesting trend to note is that in the centralized setup network traffic increases as the percentage of $beacon$ nodes increases. In the distributed setup however, the traffic decreases as the percentage of $beacon$ nodes increases. This decrease in traffic is at-

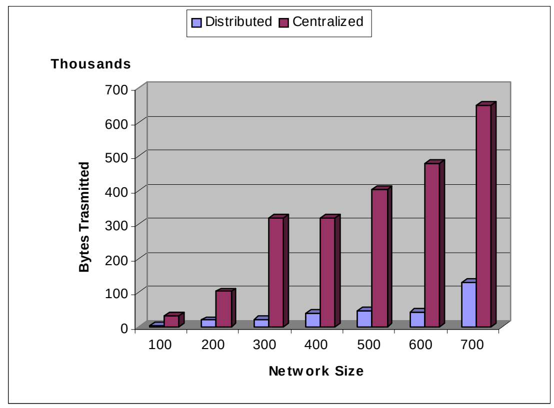

Fig. 18. Traffic in distributed and centralized implementations with 10%

tributed to the fact that most of the times the localization process can converge faster if more beacon nodes are available; hence less information exchange has to take place between the nodes. Figure 20 shows the average energy consumption per node at a transmission power of 0.24mW. This result is based on the energy characterization from the previous section. The averages where taken over all nodes so for the centralized scheme the power consumption is underestimated in this calculation since the nodes closer to the central node will burn much more energy to forward packets to and from other nodes. From this result we conclude that the useful lifetime of the sensor network will be *significantly* reduced due to the energy exhaustion of the nodes that make up the routes to the central node.

VIII. CONCLUSIONS

We have presented a new localization scheme for wireless ad-hoc sensor networks. From our study we found that the use of the ultrasound based TDoA method using ultrasound is the ideal candidate for fine-grained localization as it is less sensitive to physical effects. RF signal strength on the other hand is not suitable for fine-grained localization. Furthermore, we found that our fine-grained localization scheme should operate in a distributed fashion. This mode of operation will increase the sys-

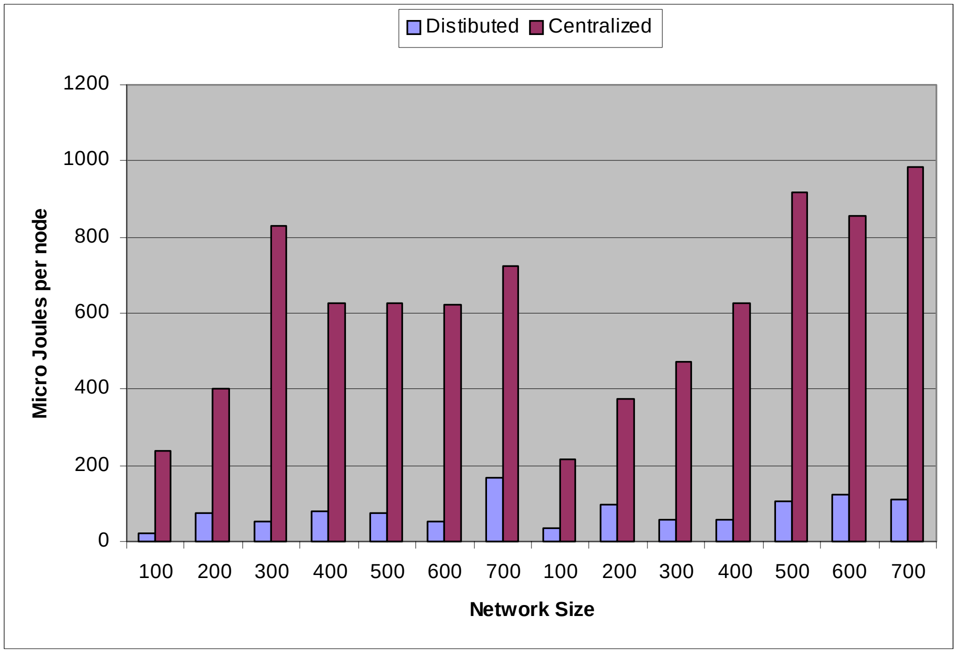

Fig. 20. Average energy spent at a node during localization, left with 10% beacons, right with 20% beacons

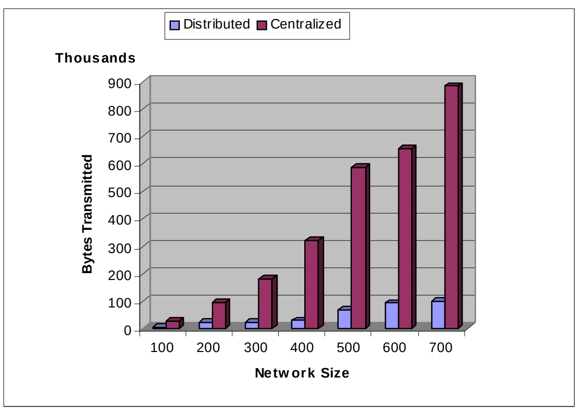

Fig. 19. Traffic in distributed and centralized implementations with 20%

tem robustness and also distribute the power cost of localization evenly among the nodes in the network. The implementation of our testbed proved to be an indispensable tool for understanding and analyzing the strengths and limitations of our approach. Although our system performed very well for our experiments, we recommend the use of a more powerful CPU on the on the sensor nodes for the following reasons. First, the CPU clock speed affects the timer granularity of the ranging subsystem. In our case the AVR microcontroller is dedicated to localization. Other applications would be hard to run on the same processor due to the very tight CPU scheduling for the ultrasound timers. Second, as we have shown in section VII, it is more power efficient to estimated node locations at each node. If the the localization algorithm is required to run at all times a faster CPU is recommended. If the network is static, the localization process can run at a very low duty cycle using the existing CPU.

The overall results of these work are very promising. Based on our experience, we are currently developing a second generation of the $Medusa$ nodes. Together with this, as part of our future work, we are considering a hybrid localization scheme that employs more that one ranging technologies. One possibility is to combine ultrasound and signal strength ranging. Since the two ranging technologies have different strengths and limitations, the one can complement the other in different scenarios.

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